

# What every scientist and engineer needs to know about low temperature plasma physics

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# Talk Outline

- Basic concepts
  - Plasma, quasi-neutrality, sheath, breakdown
  - Features of low-pressure and high-pressure plasmas
    - Non-local phenomena and plasma filaments
- Advanced concepts
  - Mechanisms of non-local electron energy distribution function (EEDF)
  - Evaporative cooling, negative conductivity, effects of Penning ionization, explosive generation of cold electrons
- Particle-in-cell Codes

# Why study low temperature plasma?

- In a low temperature plasma (LTP) the electrons are still hot enough to provide the energy required to dissociate molecules or ionize atoms, chemical radicals and ions interact with the surface of materials: that is called plasma processing.
- This is why LTP is used in most of the steps for making computer chips and memory
  - Etching
  - Deposition
  - Cleaning
- At PPPL, we use LTP for processing of materials used for next generation electronics, as well as refining the processes for making quantum diamond and diamond sensors

# Why is the fourth state of matter called plasma?

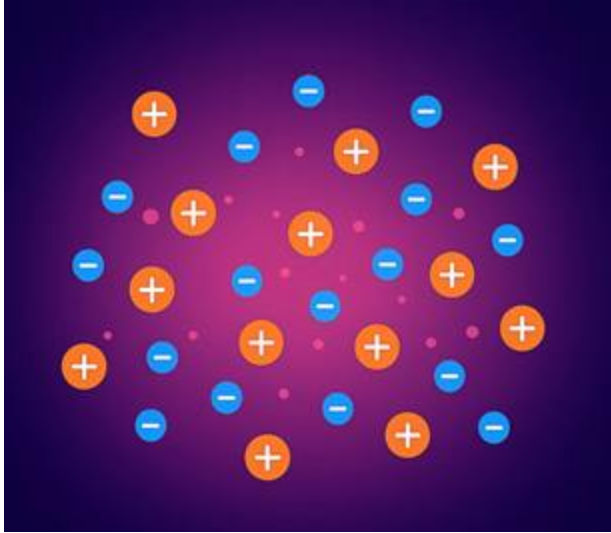
While studying electrified gases in 1927, Irving Langmuir was struck by the similarities between:

- how blood plasma transports red and white blood cells
- how an electrified gas carries electrons and ions

This led him to adopt the term *plasma* for ionized gases.



# What is plasma?



When a gas is ionized, pairs of electrons and ions are created.

The resulting plasma is like a soup of freely moving positive ions and negative electrons.

Total # of positively charged particles = Total # of negatively charged particles

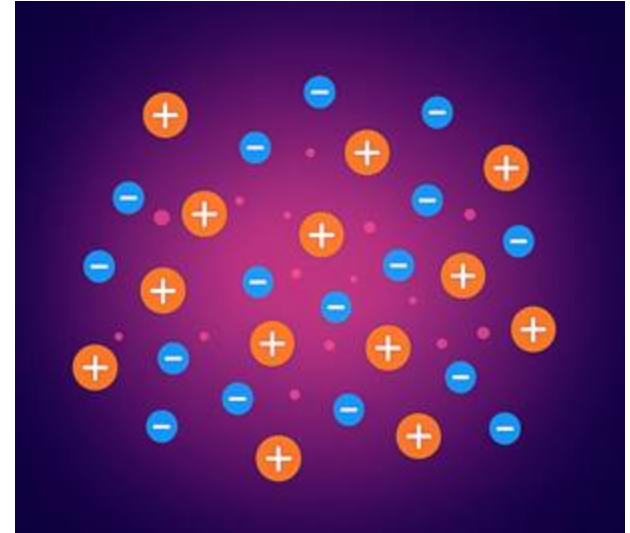
# Estimating Electrostatic Potential

$$V \approx 4\pi e^2 n_e \ell^2 \gg T_e$$

Where:

- **V** is the electrostatic potential (volts)
- **e** is the elementary charge ( $\sim 1.60 \times 10^{-19}$  C)
- **$n_e$**  is the electron number density ( $\text{m}^{-3}$ )
- **$\ell$**  is a characteristic length scale (m)
- **$T_e$**  is the electron temperature (in electronvolts, eV)

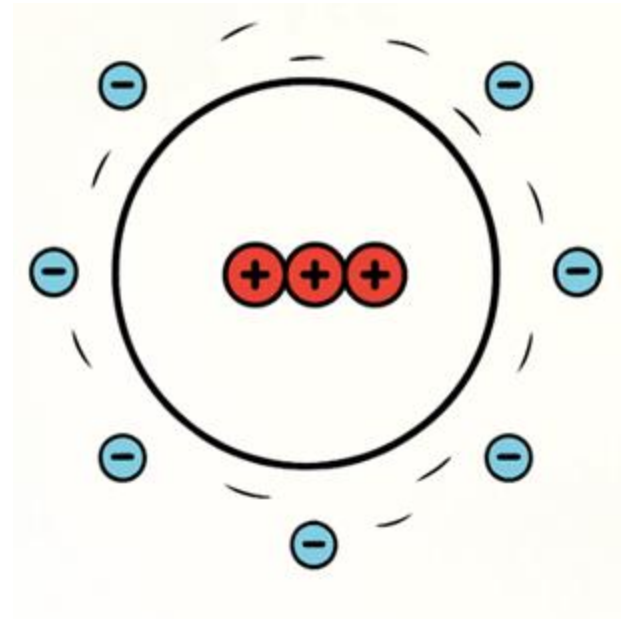
Fig 3. Plasma



# Plasma particles tend toward even charge distribution

- The individual electrons and ions can be pulled or pushed by electrical and magnetic forces.
- Electrons move faster due to their lower mass and higher mobility.
- This movement can cause a separation of charge: positive where the ions remain, and negative where electrons accumulate.

Fig 1.  $N_e$ ,  $N_i$



# What is meant by the term quasineutral?

- If part of the plasma develops a charge imbalance, electrostatic forces will move the charged particles to rebalance the plasma.
- The resulting electric field opposes further electron escape, pulling electrons back and restoring balance.
- As a result, the plasma remains **quasineutral**, meaning that over large scales, the densities of positive and negative charges are nearly equal.



# What is a sheath?

- A plasma sheath is the transition from a plasma to a solid surface.
- This thin boundary layer forms when plasma comes into contact with any material surface, such as the wall of a plasma reactor.
- When plasma encounters a surface, electrons quickly accumulate on the surface, leaving behind a region depleted of electrons.
- Once electrons are mostly depleted from this boundary, a region with only positive ions and neutrals will be formed, it is called space charge sheath or sheath for short.

**Fig 2.** Plasma reactor

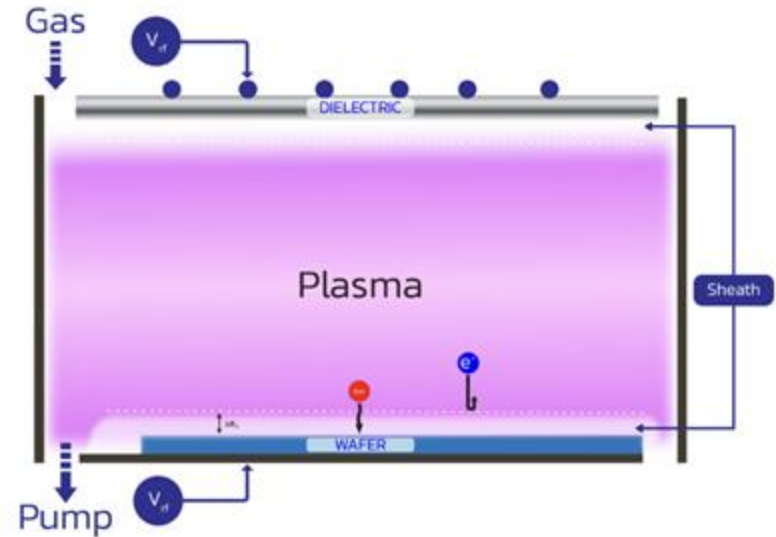


Image credit: Kruger, S. Understanding Sheath Behavior Key to Plasma Etch. Aug 11, 2022. <https://semiwiki.com/eda/tech-x/316382-kinetic-modeling-tool-for-simulating-plasma-etch-reactors/>

# What is meant by the term breakdown?

- Breakdown is the transition of an insulating gas into a conductive plasma
- It occurs when the applied voltage reaches a critical threshold (breakdown voltage)
- Gas molecules become ionized, creating free electrons and ions
- Results in rapid increase in electrical conductivity

# Breakdown vs. Maintenance Phase

- Low Density:  $4\pi e^2 n_e \ell^2 \ll T_e$ 
  - Plasma density is very small
  - Electrons flow freely and escape quickly to walls
  - High voltage required to sustain ionization process
- High Density:  $4\pi e^2 n_e \ell^2 \gg T_e$ 
  - Plasma density increases significantly
  - Electrons slowed by self-consistent electric field
  - Plasma decay controlled by slower ion motion
  - Much lower voltage needed to maintain existing plasma
- **Breakdown voltage  $\gg$  Maintenance voltage**

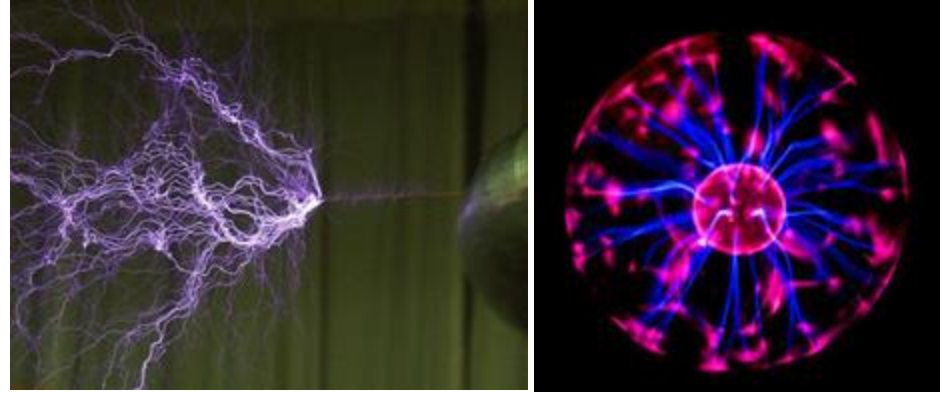
# Partially ionized versus fully ionized plasma

- If plasma is fully ionized, increasing the power leads to heating and increasing the electron or ion temperatures
- If plasma is partially ionized, additional power goes to more ionization and increasing the electron or ion density
  - The electron temperature is nearly fixed by the balance for rates of ionization and wall losses, because ionization frequency is very strong function of the electron temperature.

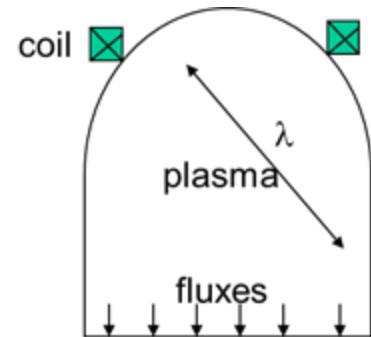
# Low pressure versus high pressure partially ionized plasma

- At high pressure ( $> 100\text{mTorr}$ ), plasma transport is reduced due to faster collisions with gas. Plasma self-organization is determined by nonlinear ionization processes often subject to filamentation.
- At low pressure ( $< 10\text{mTorr}$ ), plasma transport is dominant process, and plasma is often diffuse but nonlocal phenomena are important: the electron temperature remains high far from the sources of energy input.

**Fig 2.** Plasma filaments. Photo credit: Ian Tresman



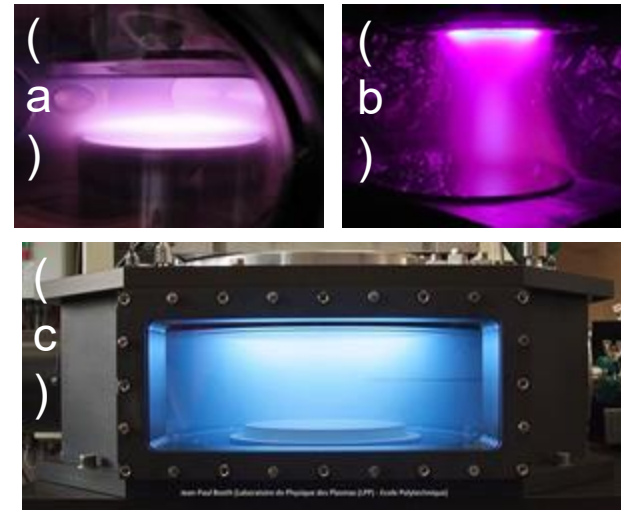
**Fig 3.** Inductive plasma coil (top) produces inductive field that penetrates chamber and maintains plasma.



# Plasma sources for semiconductor manufacturing

- Low-temperature plasmas:
  - Weakly ionized =>
  - Electrons are not in equilibrium with gas and ions
  - Higher energy electrons driving chemical processes and ionization
- Typical discharges include:
  - Capacitively coupled plasma (CCP)
  - Inductively coupled plasma (ICP)
  - Magnetrons
  - Microwave sources
  - Electron beam driven sources

**FIG.** Plasma sources for semiconductor industry, (a) CCP discharge for etching [Ruhr-University Bochum], (b) planar magnetron for sputtering and deposition [Angstrom Sciences], (c) ICP discharge for etching [Ecole Polytechnique].

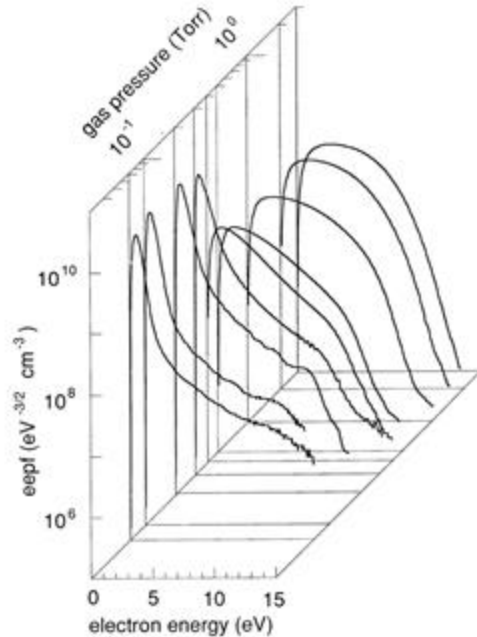


# Plasma parameters in CCP and ICP

- Plasma density,  $n = 10^9 - 10^{13} \text{ cm}^{-3}$
- Gas pressure = few mTorr
- Small degree of ionization  $< 10^{-4}$
- Electron temperature,  $T_e = \text{few eV}$
- Ion temperature,  $T_i = 0.03 \text{ eV}$
- Spatial scale = mm - m

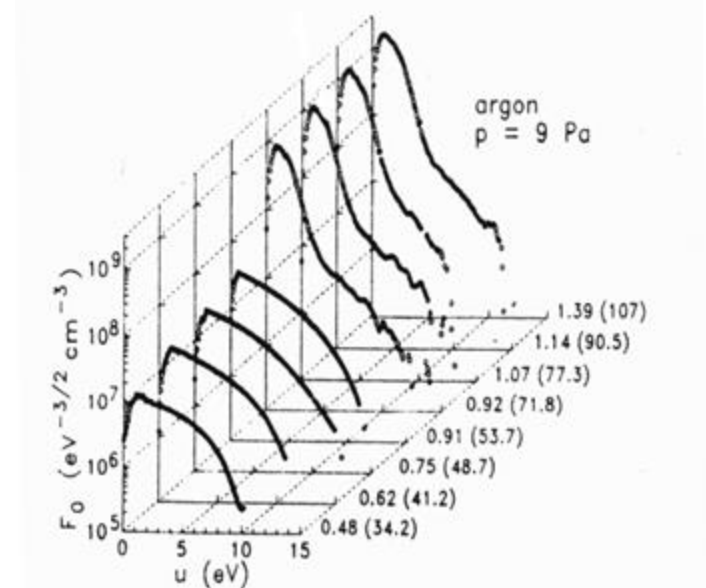
# EEDFs are non-Maxwellian

**Fig.** EEDF in capacitive discharge.



V. Godyak, IEEE TPS TPS **34**, 755 (2006).

**Fig.** EEDFs in midplane of capacitive RF discharge in Ar for constant pressure and varying RF current densities in  $\text{mA/cm}^3$  or RF discharge voltages.

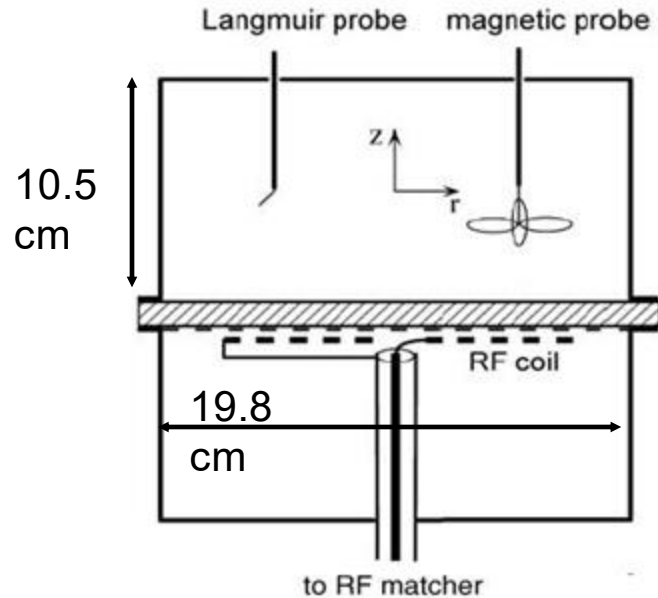


U. Buddemeier et al., APL **67**,191 (1995).



# V. Godyak's experiment is benchmark.

**Fig.** Schematic of ICP



$f=0.45-13.56$  MHz

Argon gas pressures 0.3-300 mTorr

RF power 6-400 W

**Figure 1.** Experimental discharge chamber.

V. A. Godyak, R. B. Piejak, and B. M. Alexandrovich, J. Appl. Phys. 85, 703 (1999).

# Global model: particle balance determines $T_e$

Assuming steady-state and a Maxwellian Electron Energy Distribution Function =>

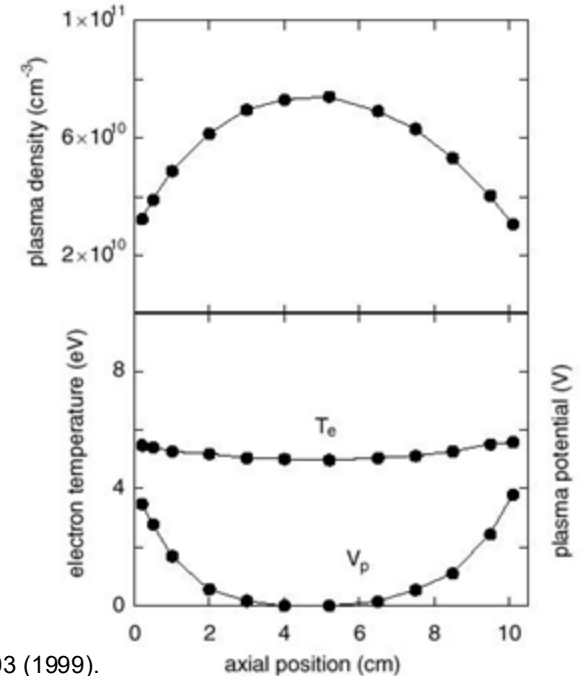
- Rate of plasma production = rate of plasma loss,
- Ionization frequency = loss frequency to the wall

$$\nu_{iz}(T_e) = \nu_{loss}(T_e)$$

$$\nu_{iz} = n_g V_T \sigma_{iz} e^{-I/T_e} \quad \nu_{loss} = \gamma C_s / L \quad C_s = \sqrt{T_e / M}$$

$$n_g V_T \sigma_{iz} e^{-I/T_e} = \gamma \frac{\sqrt{T_e / M}}{L}$$

**Fig.** Plasma profiles in ICP



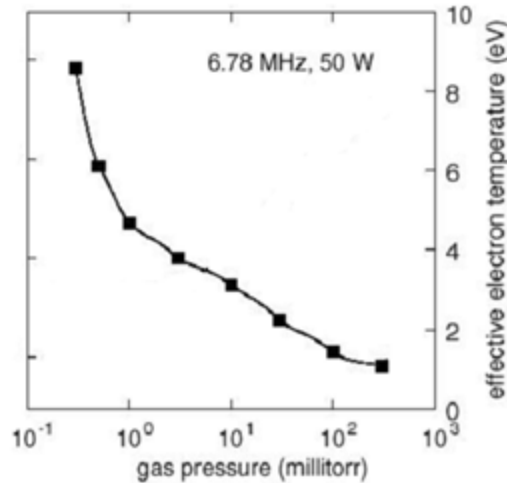
V. A. Godyak, et al., J. Appl. Phys. **85**, 703 (1999).

# Experimental data vs global model

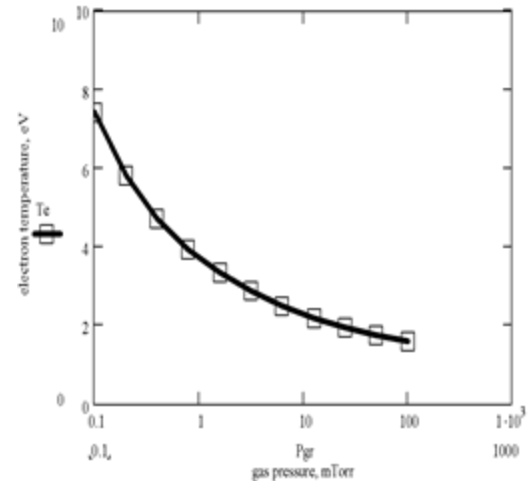
$$n_g V_T \sigma_{iz} e^{-I/T_e} = \gamma \frac{\sqrt{T_e / M}}{L} \quad \uparrow n_g L \quad \downarrow T_e$$

**Fig.**  $T_e$  as a function of gas pressure at  $f=6.78$  MHz

Experimental Data

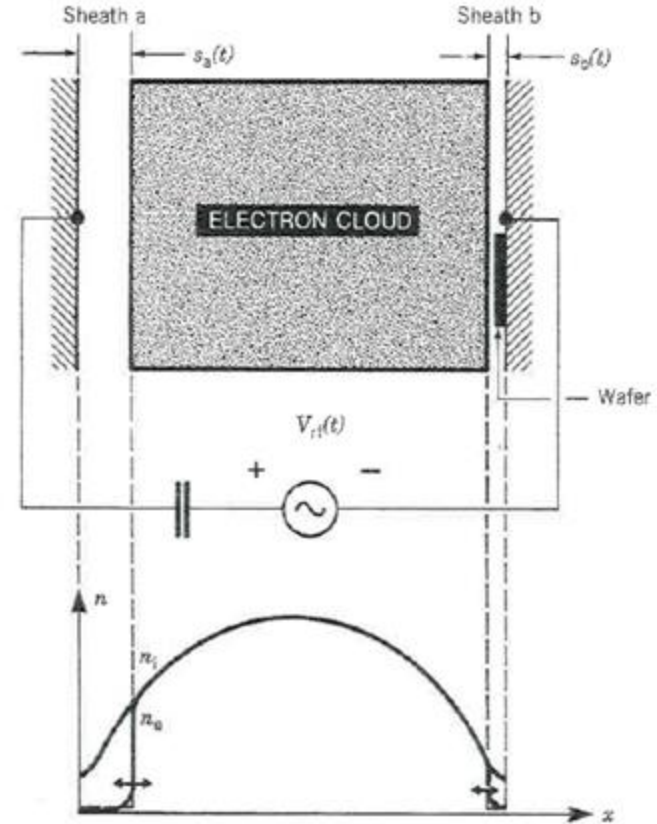


Global Model



# CCPs

- Plasma density,  $n = 10^9 - 10^{11} \text{ cm}^{-3}$
- Gas pressure = few mTorr- 1Torr
- Small degree of ionization  $< 10^{-4}$
- Electron temperature,  $T_e = \text{few eV}$
- Ion temperature,  $T_i = 0.03 \text{ eV}$
- Spatial scale = mm - m
- Applied voltage = few Volts - 10s kVolts
- Power = 10 W- 10kW
- Surface area = 100 – 300  $\text{cm}^2$



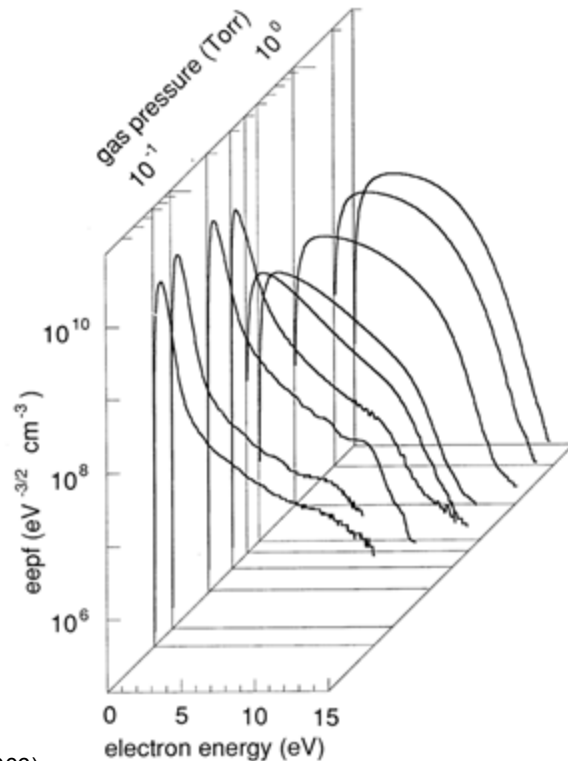
1.13. The physical model of an rf diode (after Lieberman and Gottscho, 1994).

# Discharge modeling needs to be kinetic!

Electron energy distribution functions (EEDF) are non-Maxwellian:

- Parts of the EEDF are very flexible and almost independent.

**Fig.** EEDFs for different pressures in capacitive discharge.



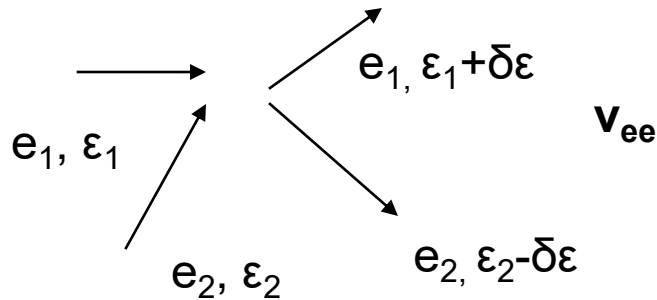
V. Godyak, IEEE TPS 34, 755 (2006).

# Mechanisms of EEDF formation

Cooling is due to energy losses in elastic and inelastic collisions.



Mixing is due to electron-electron collisions.



Electron-electron collisions drive EEDF towards a Maxwellian!  $\Rightarrow$

If  $v_{ee} \gg v^*$  or the degree of ionization,  $n_e/n_g > 10^{-4}$ , EEDF is a Maxwellian;

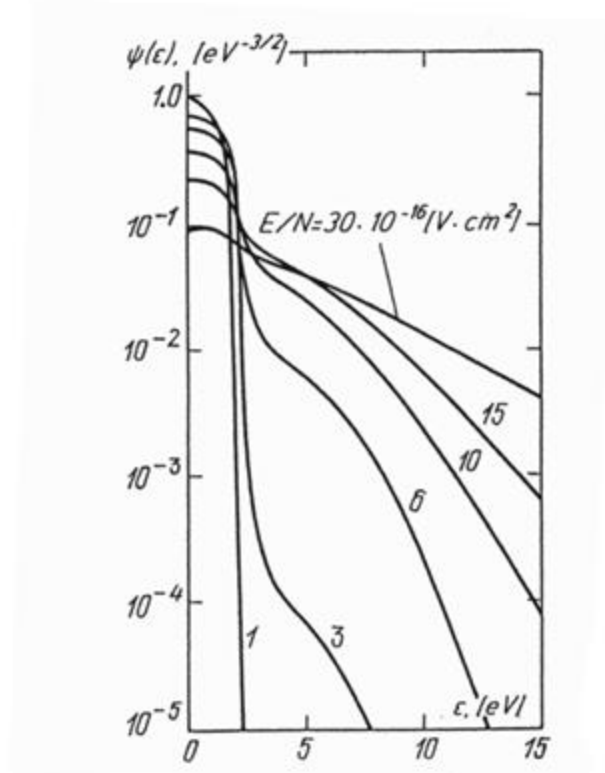
If  $v_{ee} \ll v^*$  or  $n_e/n_g < 10^{-4}$ , EEDF can have any shape.

# EEDF in nitrogen, constant electric field

1930 Druyvesteyn's EEDF for  $\lambda$  constant and  $E$ , and energy losses in elastic collisions.

$$f \sim \exp(-\varepsilon^2/\varepsilon_0^2) \quad \varepsilon_0 = eE\lambda\sqrt{M/m}$$

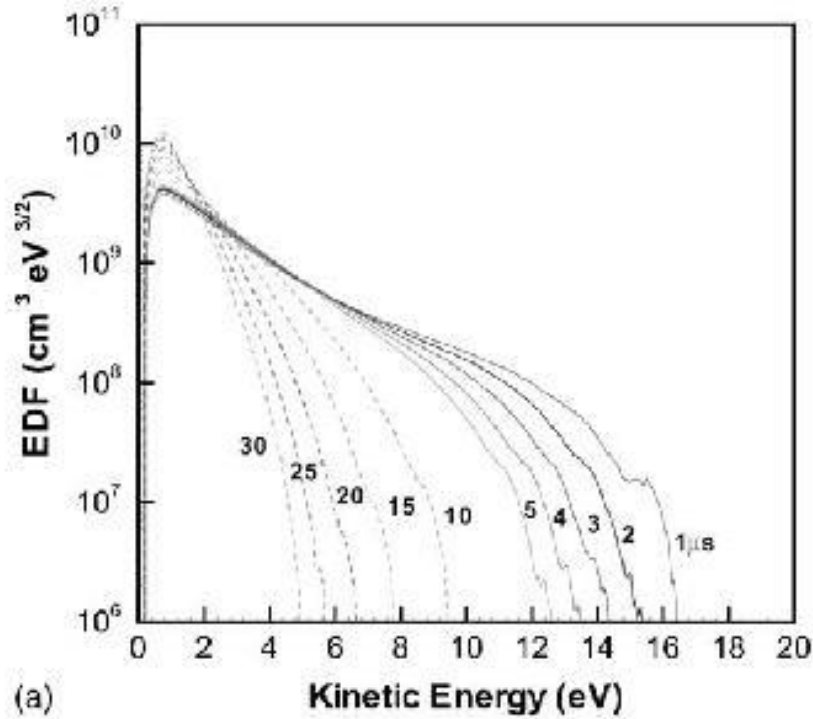
**Fig.** EEDF in nitrogen in given  $E/N$



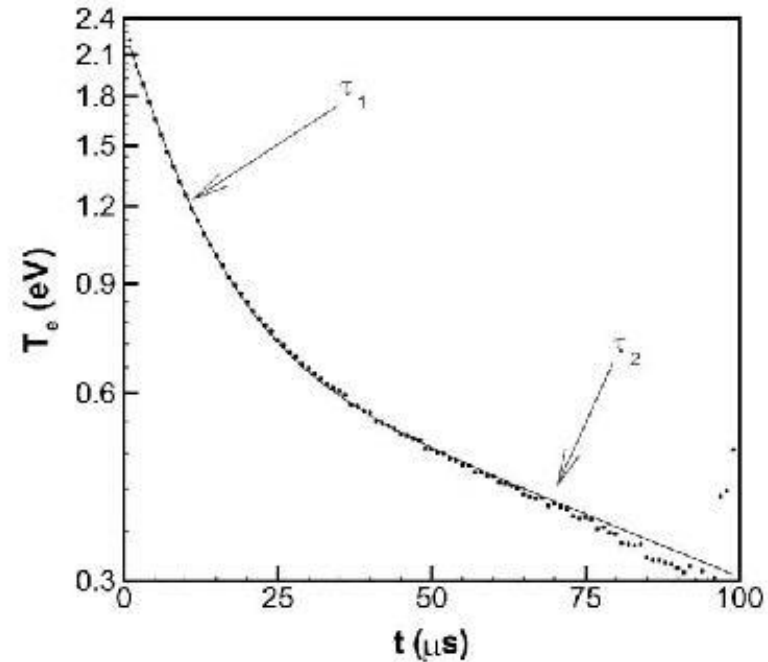
N.L. Alexandrov, et al. Sov. J. Plasma Phys. 1978

# Evaporative Electron Cooling in Afterglow

**Fig.** EEDF in argon as a function of time

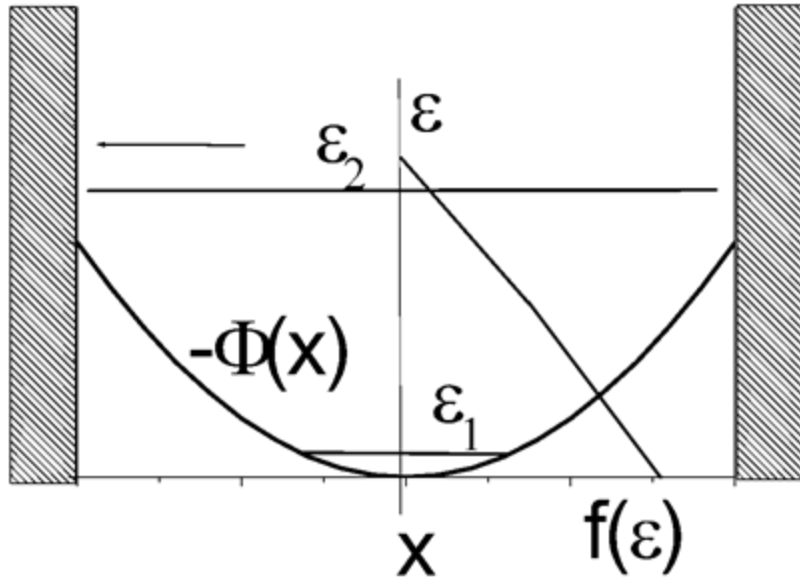


**Fig.** Effective  $T_e$  as a function of time





# Evaporative Electron Cooling in Afterglow



**Electron temperature can cool much below room temperature, observed up to 30K Biondi (1954)**

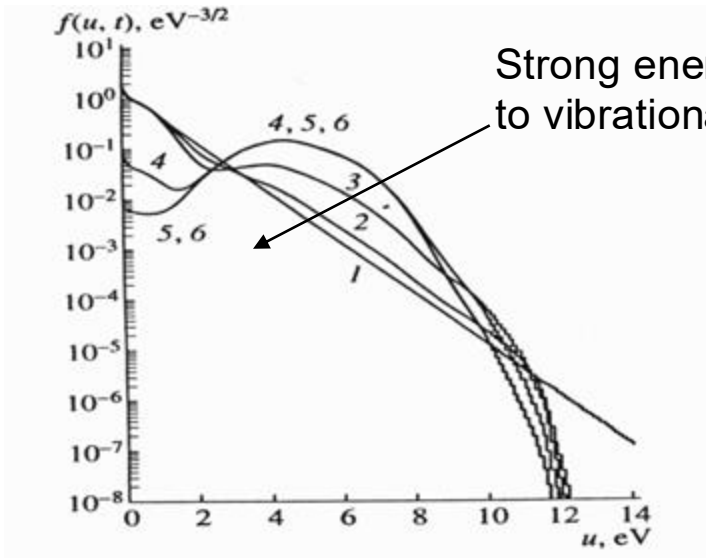
In non-uniform plasmas low-energy electrons are trapped, where as high-energy electrons can leave. =>

$T_e$  reduces when plasma decays

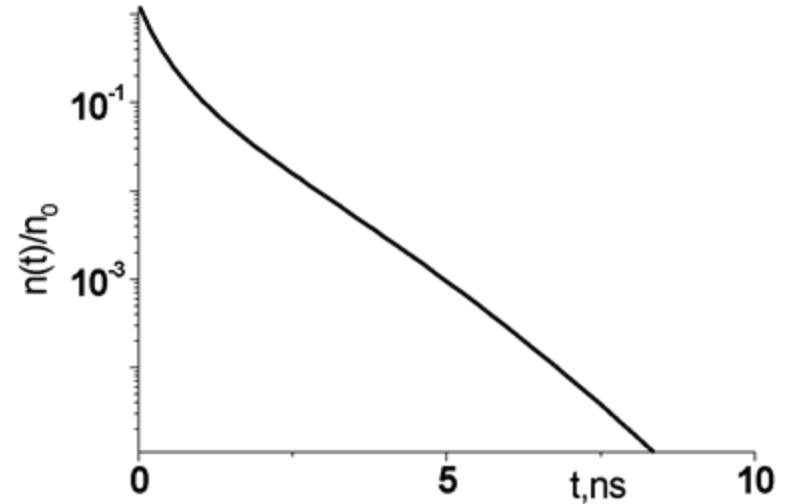


# EEDF in decaying plasma

**Fig.** EEDF afterglow Ar:NF<sub>3</sub>.  $E/N=2 \cdot 10^{-17} \text{Vcm}^2$   
1, 2, 3, 4, 5, 6. 0, 0.25, 1, 3, 5, 10 ns.



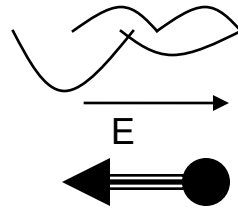
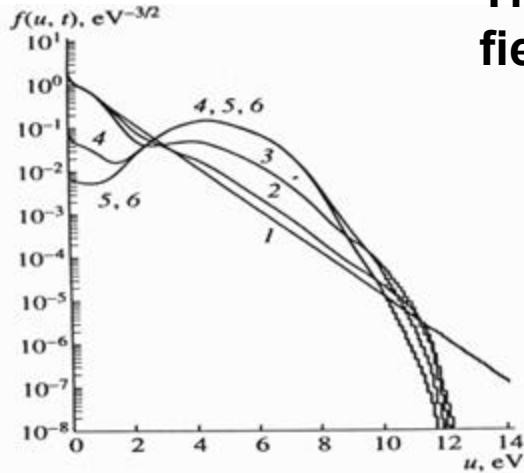
**Fig.** Density as function of time in decaying plasma



N.A. Dyatko, et al. Plasma Phys. Rep. 1998

# Non-monotonic EEDF yields negative plasma conductivity!

The total electron current is opposite to the electric field



electron current

$$\mu = -\frac{2e}{3m} \int u^{1/2} \lambda \frac{df(u)}{du} du < 0$$

$$\frac{df_0}{dt} = \frac{1}{\sqrt{e}} \frac{\partial}{\partial e} \left( \frac{1}{\sqrt{e}} \frac{\partial f_0}{\partial t} \right)$$

# Effects of Penning ionization in afterglow

In the afterglow, electrons cool rapidly to  $T_e \sim 0.2$  eV.

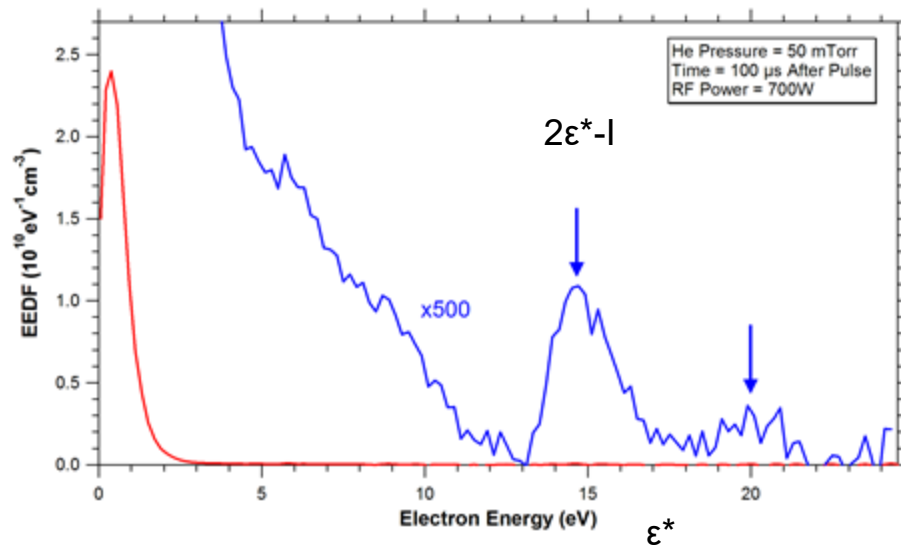
However, a small amount of fast electrons with energy few eV arise from slowly decaying metastables, due to, e.g., the Penning ionization



$$\text{He } \epsilon^* = 20 \text{ eV } I = 25 \text{ eV}$$

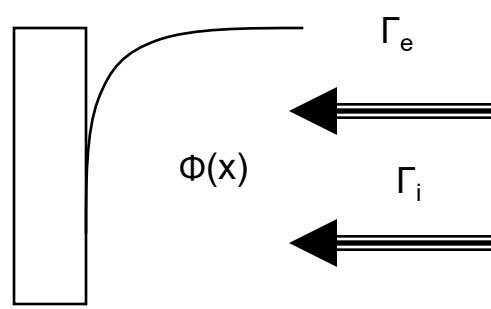
$$2\epsilon^* - I = 2 \times 20 - 25 = 15 \text{ eV}$$

**Fig.** Experimental EEDF in He afterglow,



V.I. Demidov et al, Phys Rev. Lett. (2005)

# Effect of a Small Population of Fast Electrons On Wall Potential



$$\Gamma_e = n \sqrt{\frac{T_e}{2\pi m}} e^{-\phi/T_e}$$

$$\Gamma_i = n \sqrt{\frac{T_e}{M}}$$

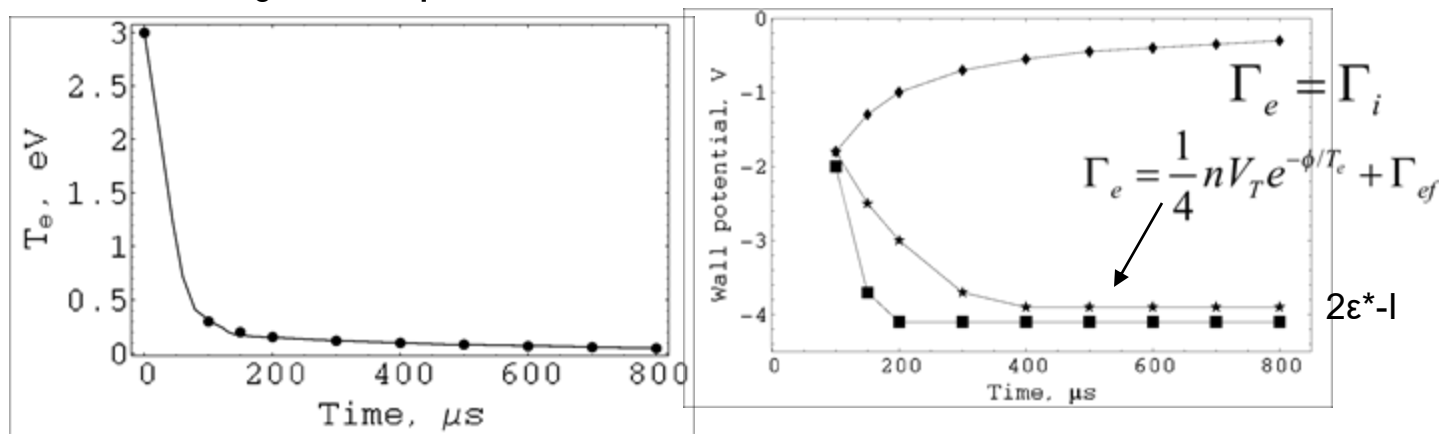
$$\Gamma_e = \Gamma_i$$

$$e\phi_p = T_e \ln \left( \sqrt{\frac{M}{2\pi m}} \right) \sim 5T_e$$

Relies on the assumption of a Maxwellian electron energy distribution function. Small flux of energetic electrons with  $\Gamma_{ef} > \Gamma_i$  can strongly modify sheath.

# Effect of a Small Population of Fast Electrons On Wall Potential

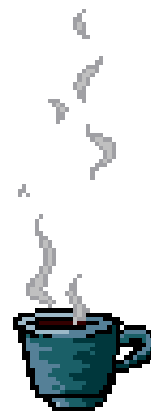
**Fig.** Afterglow in 0.2Torr Xe. Left - exp.  $T_e$ . Right - the wall potential: stars – exp. data, diamonds -  $5T_e$  and squares – calculations.



V.I. Demidov, et al, PRL **95**, 215002 (2005).

Near-wall potential drop can be much greater than  $T_e \Rightarrow$

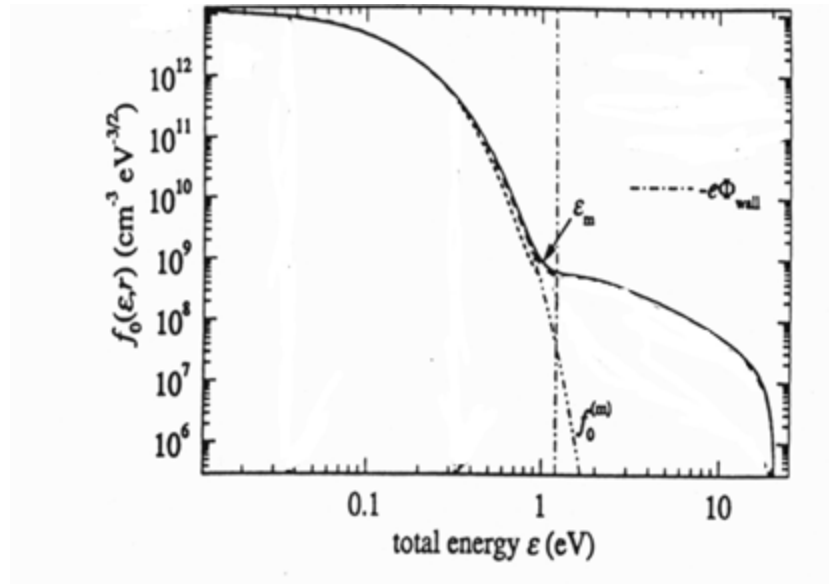
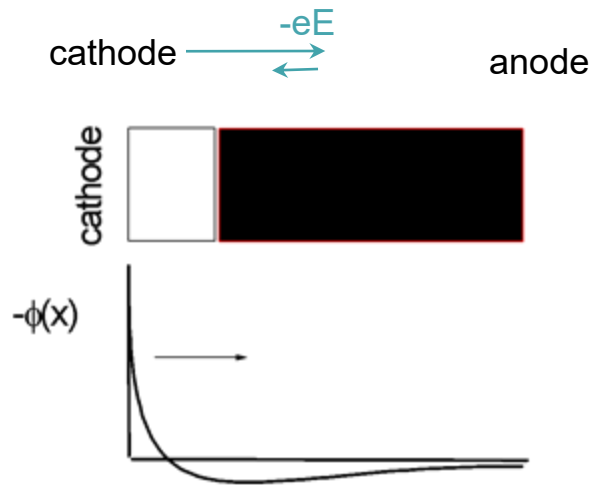
This large wall potential can lead to an increase in electron density, diffusion cooling can be eliminated.



# The Abrupt Formation of Large Population of Cold Electrons

- Large population of cold electrons is typical in various glow discharges.
- “Paradoxical” electron cooling with power increase.
- Plasma density jumps with power increase.

# Cold Electron Formation the Negative Glow of the DC discharge



## Cathode fall

He 3.5 Torr, 0.85mA/cm<sup>2</sup>, 260V, 0.62cm

Exp.:  $T_e=0.12\text{eV}$ , E.A. Hartog et. al., PRL 1989

Model: R. Arslanbekov, A. Kudryavtsev PRE 1998

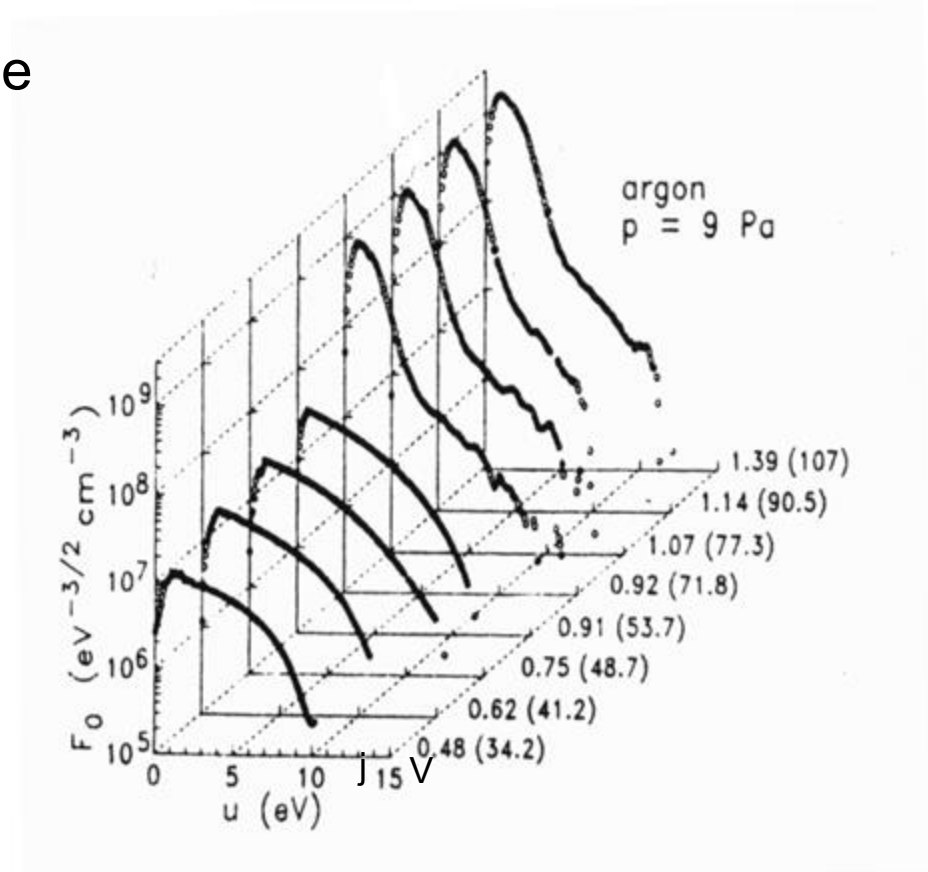


# Explosive Generation of Cold Electrons In Capacitive Discharge

EEDF modification with discharge current

Argon, 13.56MHz, 6cm, 9Pa

U. Buddemeier et.al., APL (1996)

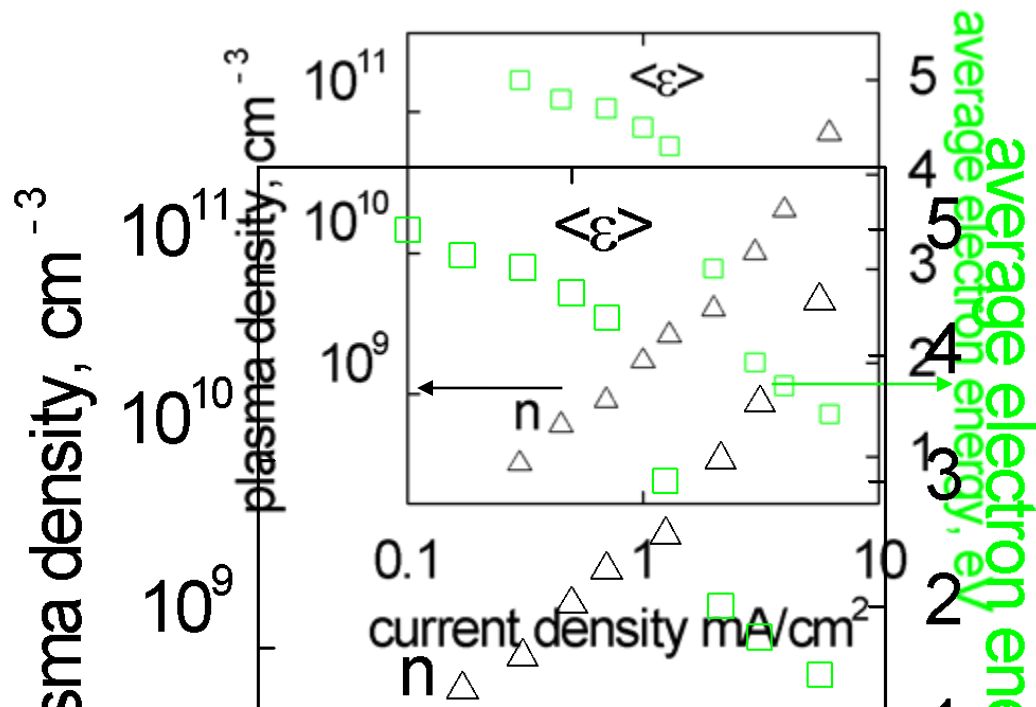
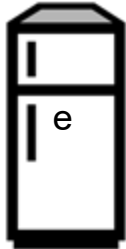


# “Electron Refrigerator”

Plasma parameters evolution with current

Argon, 13.56MHz, 6.7cm, 0.1torr

Symbols: exp. V. Godyak *et. al.* Plasma Sci. & Technol. (1992)



# Nonlocal, nonlinear electron kinetics

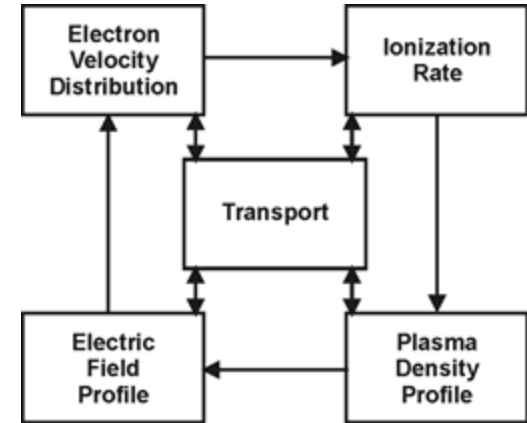
Formation of non-Maxwellian, non-uniform,  $f(v,r,t)$  in self-consistent electric field strongly coupled to plasma density profile through ionization and transport for realistic discharge plasmas.

$$\left[ \frac{\partial}{\partial t} + (\mathbf{v} \nabla) - e\mathbf{E} \frac{\partial}{m \partial \mathbf{v}} \right] f = \sum_k \left[ \nu_k^* \frac{\sqrt{u'}}{\sqrt{u}} f(w + w_k^*) - \nu_k^* f \right] + St_{ee},$$

$$\frac{\partial n_i}{\partial t} + \nabla(\mathbf{v} n_i) = \nu_{iz} n_i, \nu_{iz} n_i = \int_I^\infty f(u) v \sigma_{iz} d\mathbf{v}$$

$$\nabla \mathbf{E} = 4\pi e(n_i - n_e)$$

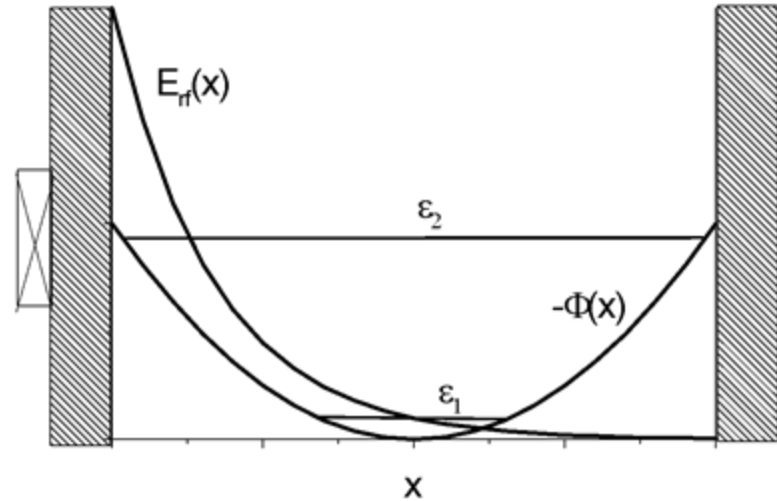
$$M_i \left( \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \nabla) \mathbf{v} \right) = e\mathbf{E} - \frac{\nabla p_i}{n_i} - (\nu_{ia} + \nu_{iz}) \mathbf{v}$$



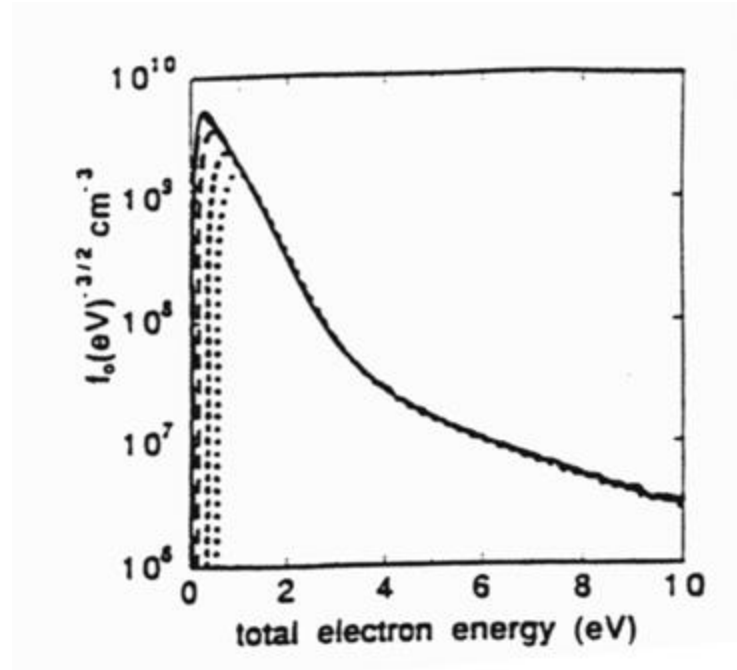
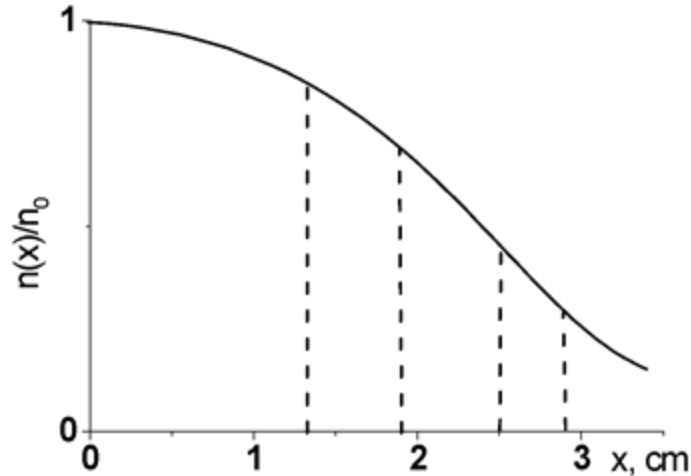
Igor D. Kaganovich et al, Phys. Rev. Lett. 1999, 2000, 2002, 2009, 2012, 2013; Valery Godyak et al, Phys. Rev. Lett. 1990, 1992, 1996, 1998, 1999.

# Formation of Cold Electrons: Explanation

- **Necessary conditions:**
  - Non-locality
  - Non-linearity



# NONLOCAL EEDF IS A FUNCTION OF TOTAL ENERGY

$$\varepsilon = \mathbf{M}\mathbf{v}^2/2 - e\phi(\mathbf{x})$$


Experimental EEDF's at different positions, capacitively coupled RF discharge in argon  
0.03 Torr, 13.56 MHz;

Exp.: V. A. Godyak and R. B. Piejak, APL 1993

PIC: V. A. Schweigert, *et al*, Appl. Phys. Lett. 69, 2341 (1996).

# Kinetic Equation Is Averaged over Fast Electron Bouncing in Potential Well

$$-\frac{d}{d\varepsilon}(D_\varepsilon + \overline{D_{ee}})\frac{df_0}{d\varepsilon} - \frac{d}{d\varepsilon}\overline{V_{ee}}f_0 = \sum_k \left[ \overline{v_k^*(u + \varepsilon_k^*) \frac{\sqrt{(u + \varepsilon_k^*)}}{\sqrt{u}} f_0(\varepsilon + \varepsilon_k^*) - \overline{v_k^*} f_0} \right],$$

**Energy diffusion  $D_e$  coefficient is a function of the rf electric field  $E_y$  and the plasma potential  $\phi(x)$ .**

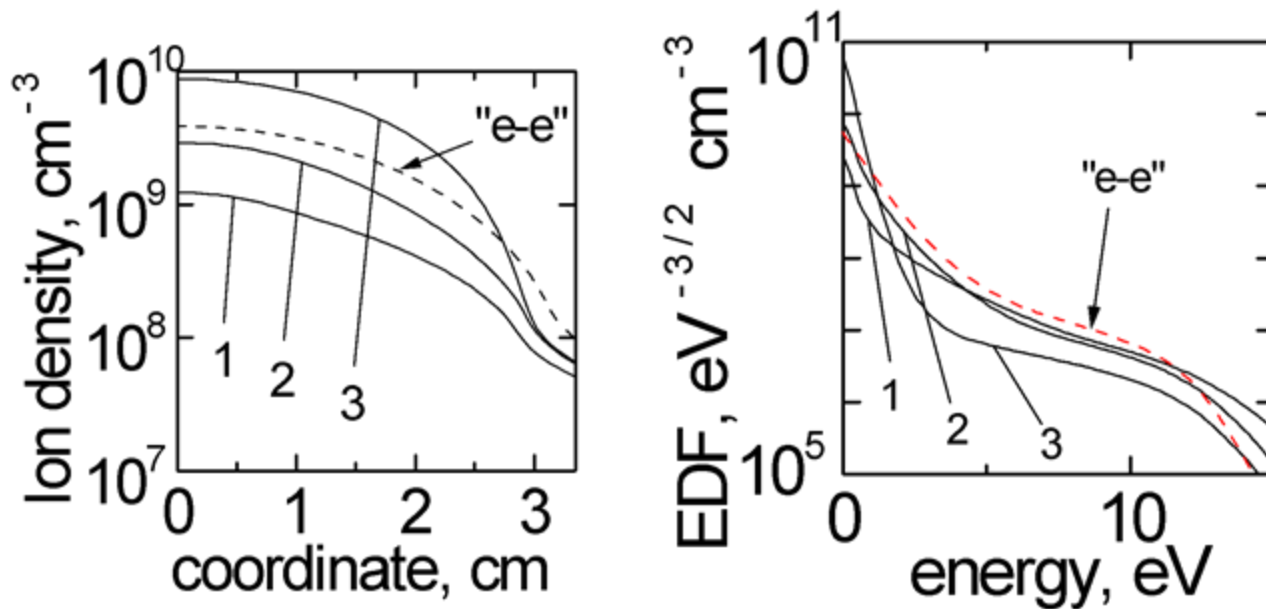
$D_{ee}$   $V_{ee}$  are from the electron-electron collision integral,  
 $v^*$  is inelastic collision frequency,  
 upper bar denotes space averaging with constant total energy.

# MODELING: ABSENCE OF STEADY STATE

The EDFs and ion density profiles at three subsequent times

Solid lines: e-e collisions are ignored. (1)  $t=0\text{ms}$ , (2)  $t=0.34\text{ms}$ , (3)  $t=2.24\text{ms}$ .

Dashed lines: with e-e collisions



# Particle-in-cell codes (PIC)

## Preferred Computational Tool - PIC

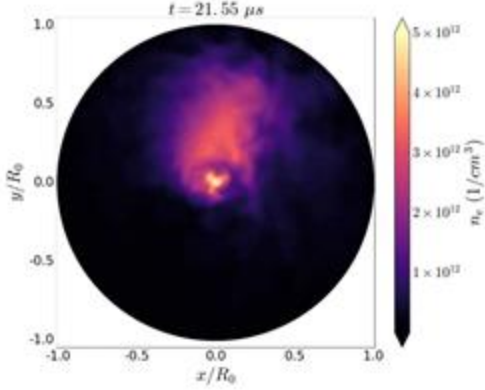
Parallel, implicit particle-in-cell code with energy corrections and noise control is a powerful and versatile tool.

- Ease of coding
  - Modular, “easy” for parallelization
  - Modern clusters and multi cores PCs are cheap
- 
- PPPL codes: EDIPIC-2D and LTP-PIC-3D



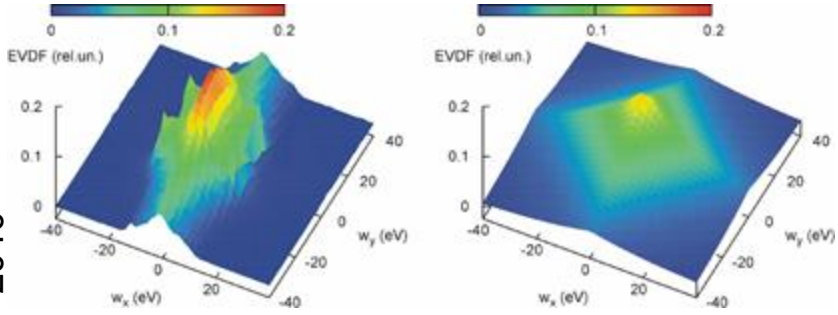
# Selected PPPL computational and scientific accomplishments using PIC

Self-organized structures in magnetized plasmas



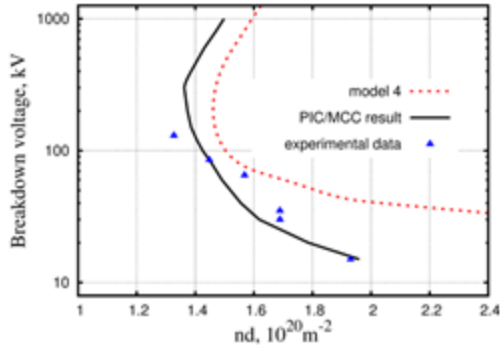
D. Sydorenko, I. Kaganovich, et al. 2010

Non-Maxwellian Electron Velocity Distribution in Hall thrusters



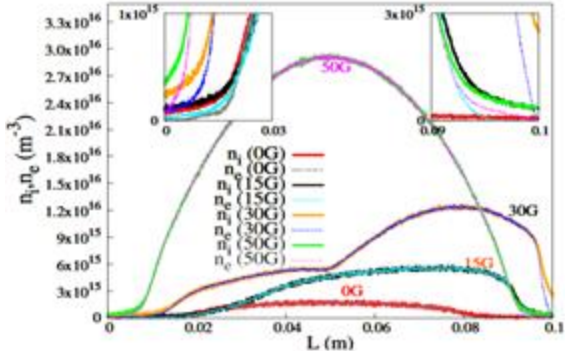
Breakdown in high voltage devices

L. Xu, et al. 2019



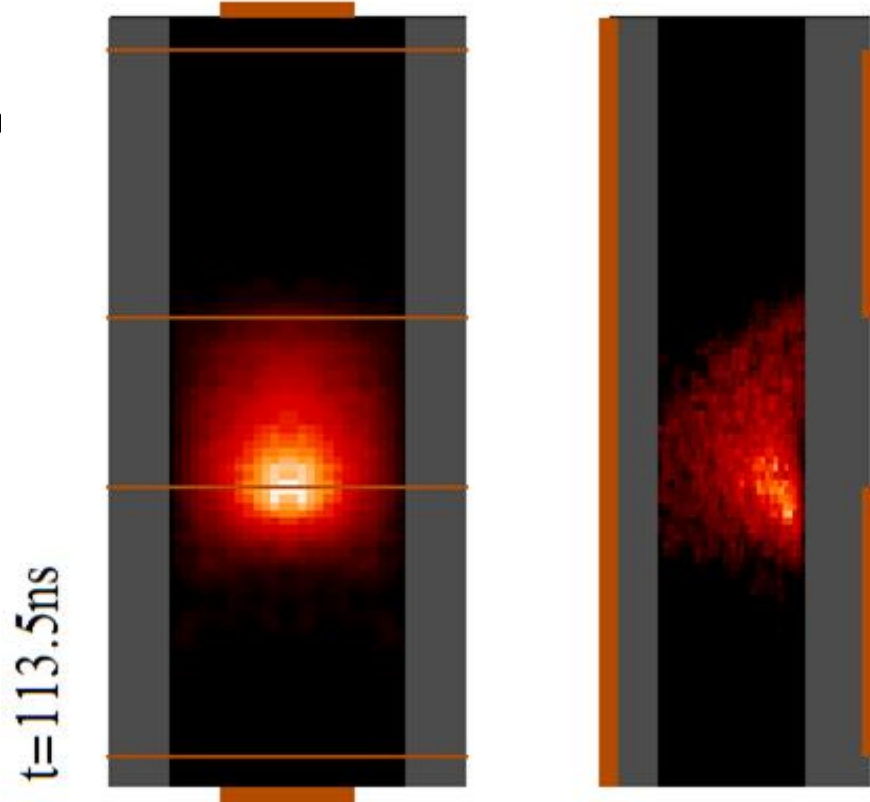
S. Sharma, et al. 2019

Effect of weak magnetic field on radio-frequency discharge



# 3D code for plasma panel (plasma TV) modeling

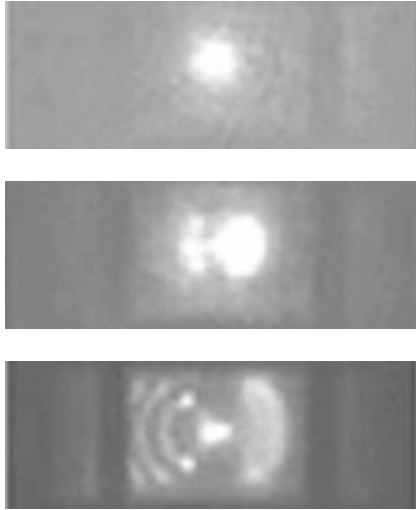
- Up to 250 million of ions and electrons
- Up to  $150 \times 150 \times 150$  meshes



V. N. Khudik, et al., IEEE Trans Plasma Sci. 33, 510 (2005).

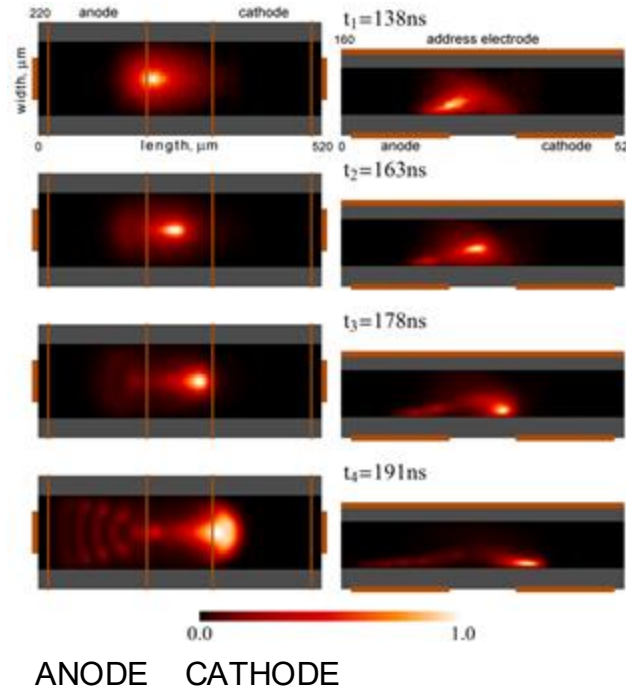
# Complex structure of discharge in plasma TVs

Experiment



Top view

Side view



**Credit:** V. N. Khudik, et al., IEEE Trans Plasma Sci. **33**, 510 (2005).

# Brief Overview of PPPL Particle-in-cell Codes (PIC)

- **EDIPIC-2D**

- 2D Cartesian and cylindrical geometry.
- State-of-the art collision models. Plasma surface interaction and circuit models. Poisson solver. Abundant diagnostics. Validated by numerous benchmarks.
- Inner objects, implicit algorithms, electrostatics and electromagnetics.
- Open source with many users.

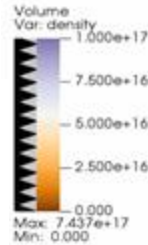
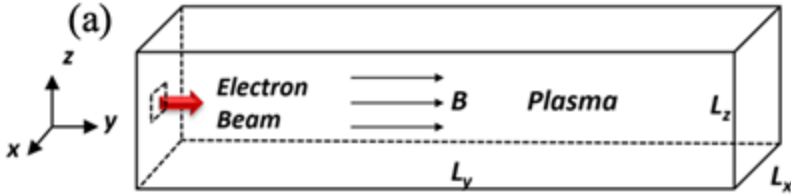
- **LTP-PIC-3D**

- Full 3D electrostatic PIC code designed to scale well as required for computationally-demanding high performance computing (HPC) simulations. Hybrid architecture for CPU+GPU.
- A lot of interest from potential users.

# LTP-PIC Example – 3D spoke in a Penning discharge

- Fully kinetic 3D simulations [1] of a Penning Discharge [2,3] **J. Chen, A.T. Powis, et al. (2023)**

**Fig.** Schematics of simulation set up



**Fig.** Movie of 3D density profile



[1] J. Chen, A. Powis, et al (2023)

[2] E. Rodriguez et al. Phys. Plasmas **26**, 053503 (2019)

[3] A. Escarguel, The European Physical Journal D **56**, 209 (2010)

# LTP-PIC Example – 3D Hall thruster channel simulations

Axial-azimuthal slice at fixed radius of a Hall thruster channel [Villafana 2023]

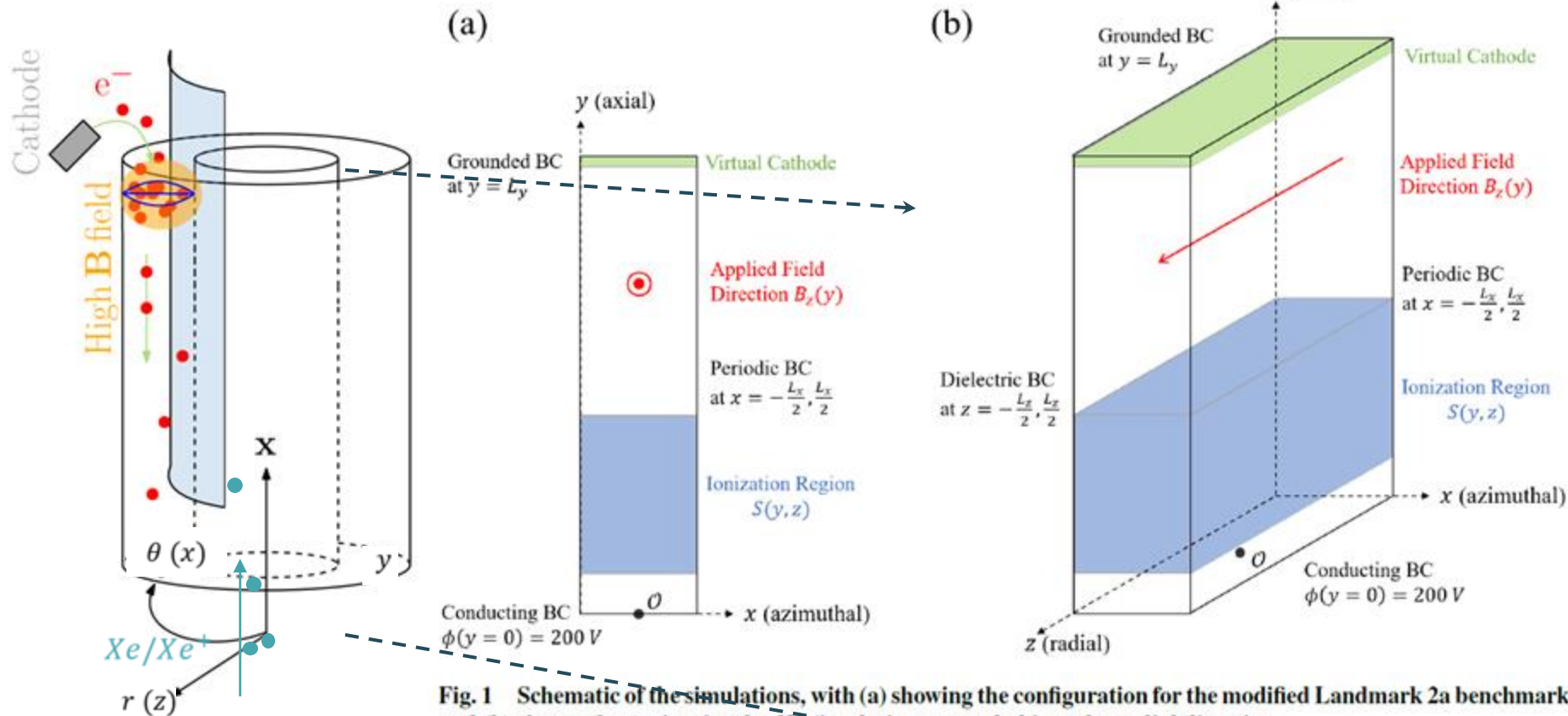
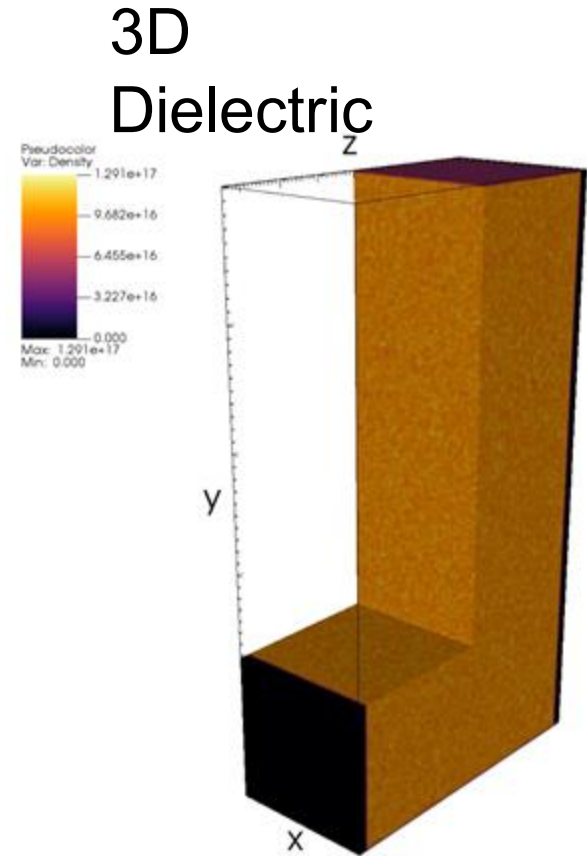
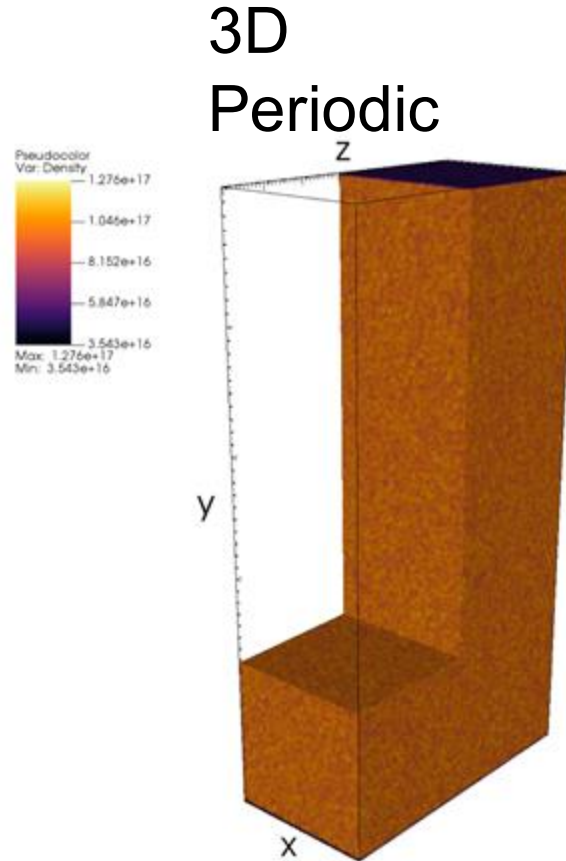
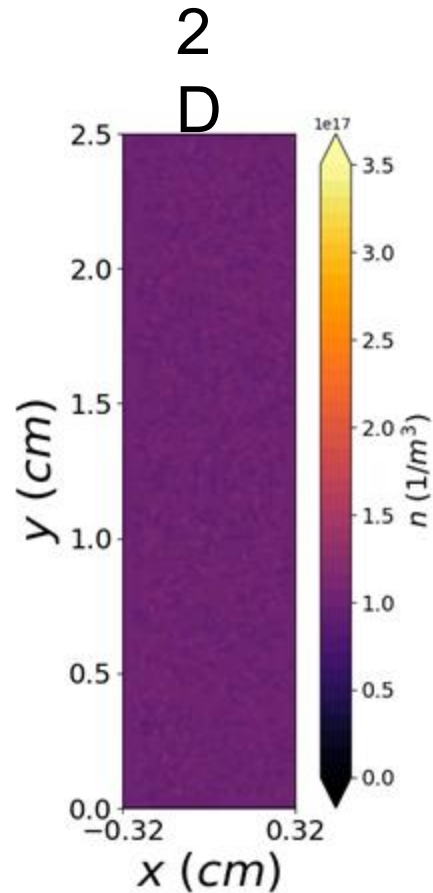


Fig. 1 Schematic of the simulations, with (a) showing the configuration for the modified Landmark 2a benchmark and (b) the configuration for the 3D simulations extended into the radial direction.

# LTP-PIC Example – 3D Hall thruster channel simulations

## Different electron transport in 3D vs 2D. Influence of boundary conditions



# Advanced Topics not Covered

- Simulation Tools
  - Descriptions of PPPL Codes
  - Algorithms for Reduced Cost Kinetic Simulations
  - High Performance Computing
  - Issues with Numerical Noise
- Physics Studies
  - Breakdown in narrow gaps, high voltage devices, studies of hollow cathode
  - Electron-Beam and Ion-Beam Interaction with Plasma
  - CCP, ICP + effects of external magnetic field
  - Penning Discharge, Hall Thrusters
- Surface Chemistry Studies for Etching and Deposition
- **Acknowledgment** to Rachel Kremen for preparing introduction slides.

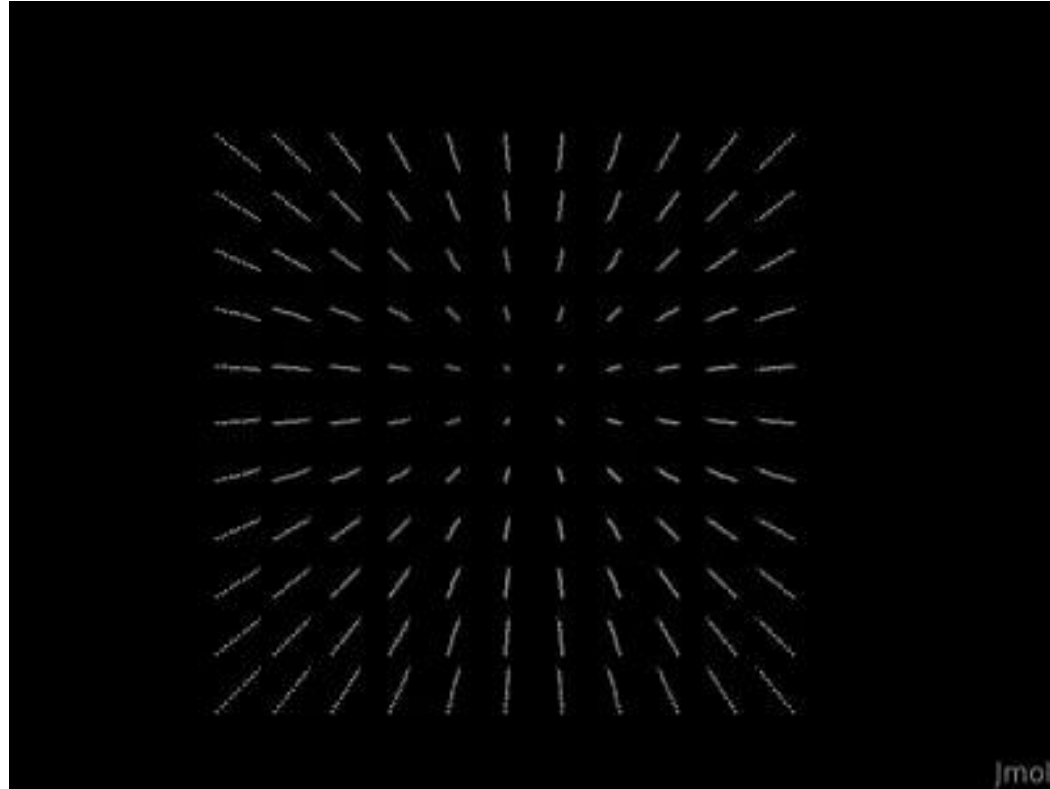


# Back up Slides

# Synthesis of Carbon and Boron Nitride Nanotubes

Modeled Carbon  
and Boron Nitride  
Nanotubes  
synthesis in arcs  
and torches.

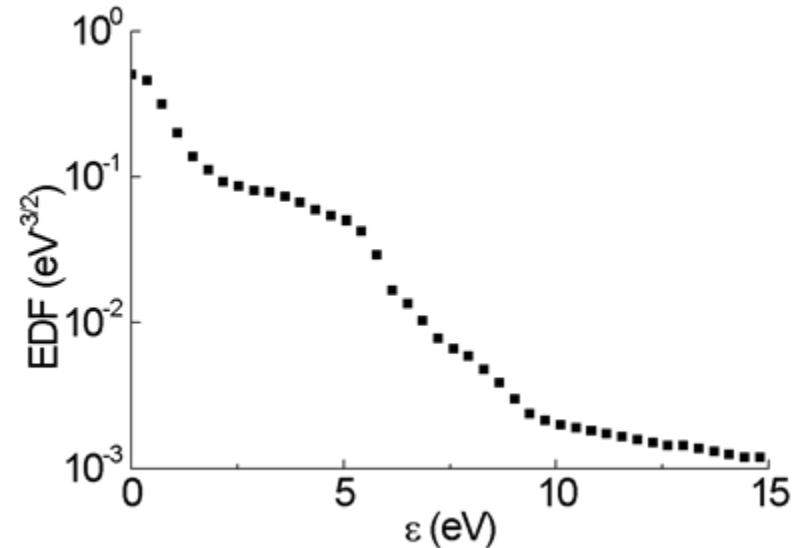
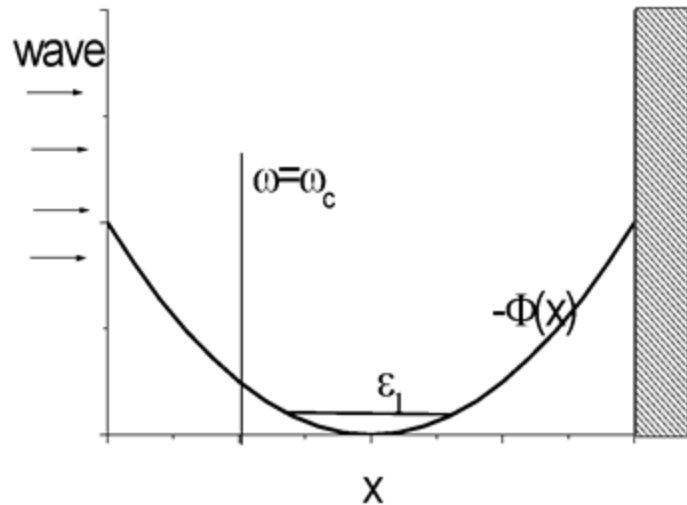
Example: Carbon  
fullerene formation  
during rapid cooling  
of carbon vapor  
using DFTB+ codes.  
L. Han et al.



# Cold Electrons in ECR Discharge

Electron cyclotron resonance discharge  $N_2$  1mTorr,  $0.14W/cm^2$ , 50cm

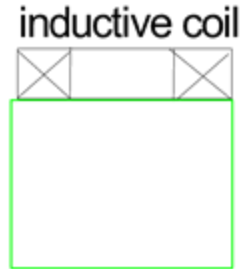
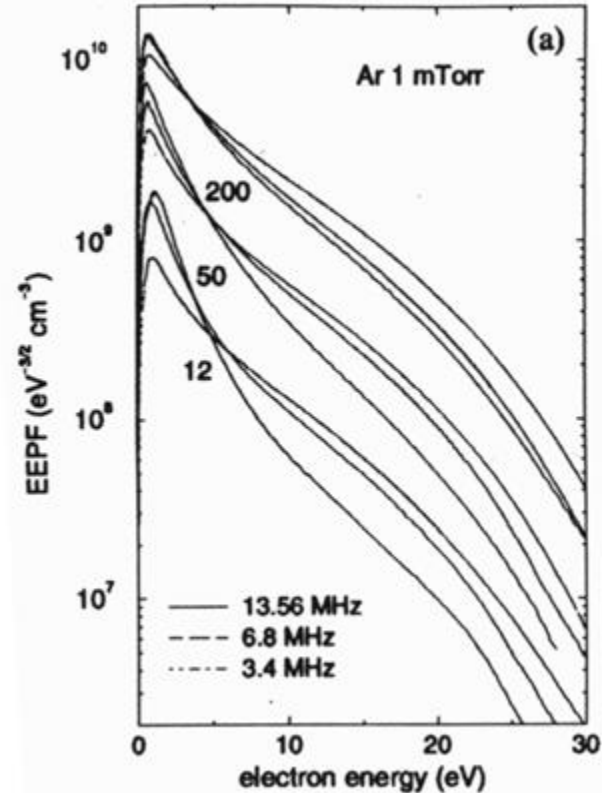
- Exp.: N. Bibinov, *et al.*, Rev. Sci. Ins. 1998, 2004
- Model.: I. Kaganovich *et al.* PRE 1999



# Cold Electrons: ICP

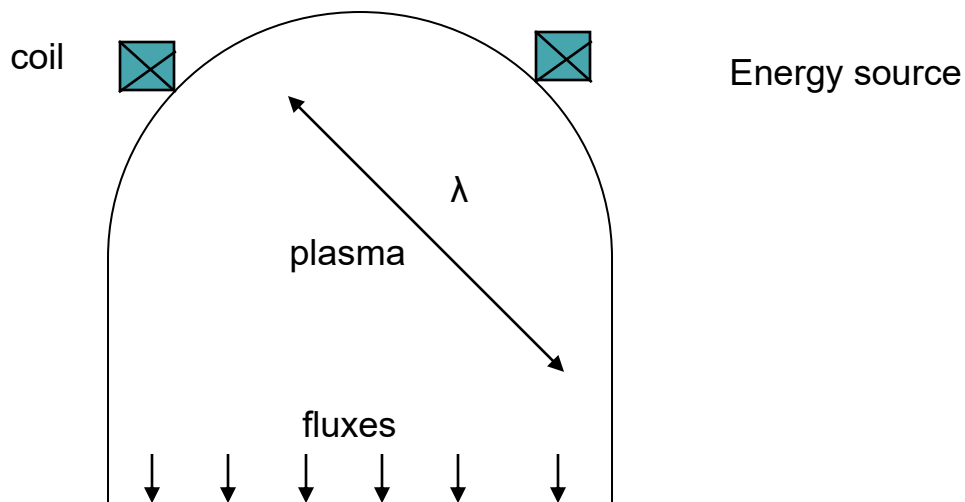
## Inductively coupled discharge

- Argon 1mtor,
- Pancake geom. 20x10.5cm
- V A Godyak & V I Kolobov  
PRL 1998

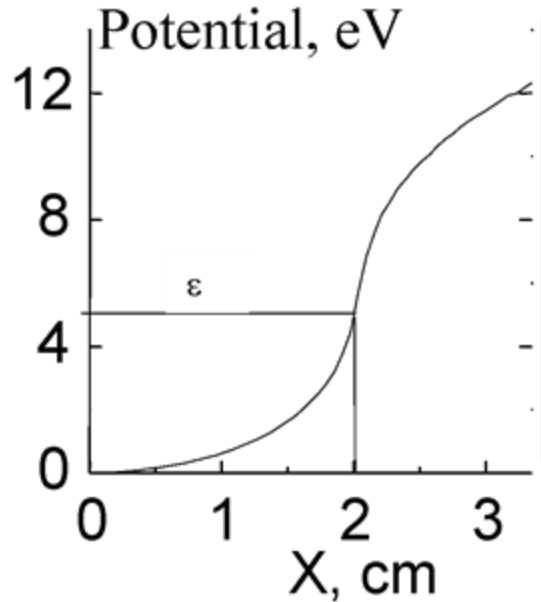


# Nonlocality Is Important For Many Plasma Applications

Electron energy relaxation length is large; this allows remote plasma handling via nonlocal electron energy distribution function (EEDF).



# Nonlocal Approach - Averaging Over Fast Electron Bouncing



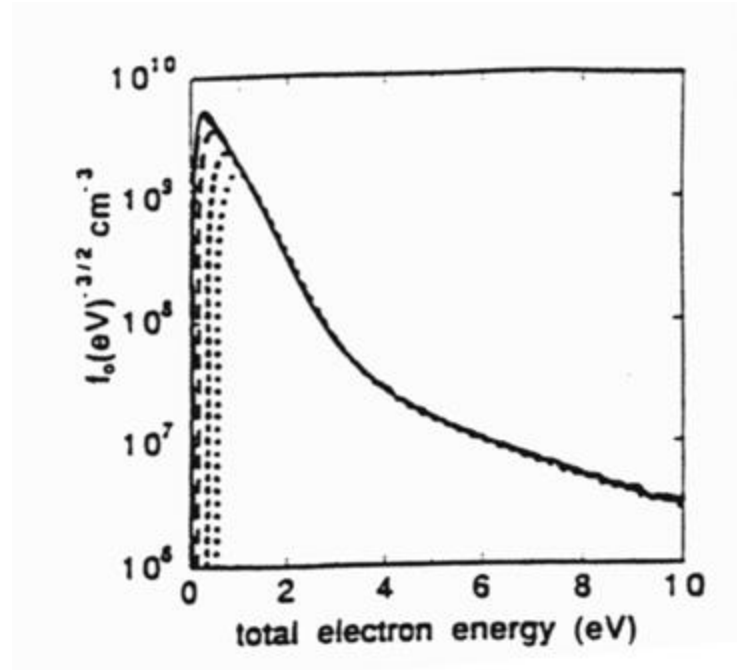
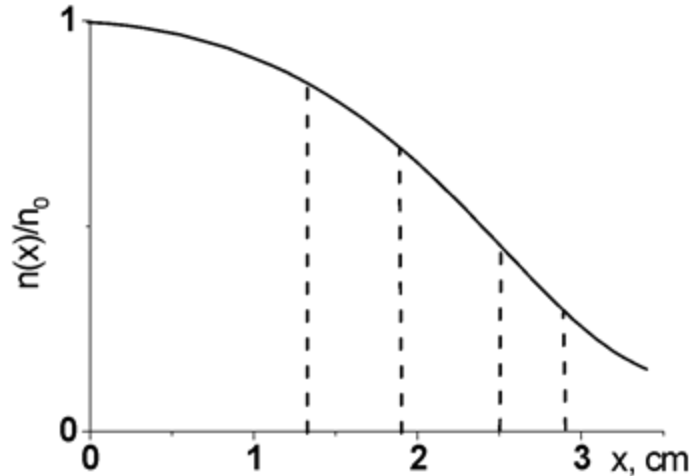
$$\varepsilon = \frac{mv^2}{2} + e\Phi(\vec{r})$$

$F(\varepsilon)$  is the function of total energy only

I.B. Bernstein, T. Holstein, Phys. Rev. 94, 1475 (1954).  
L.D. Tsendin, Sov.Phys. - JETP, 39, 805 (1974).



# NONLOCAL EEDF IS A FUNCTION OF TOTAL ENERGY

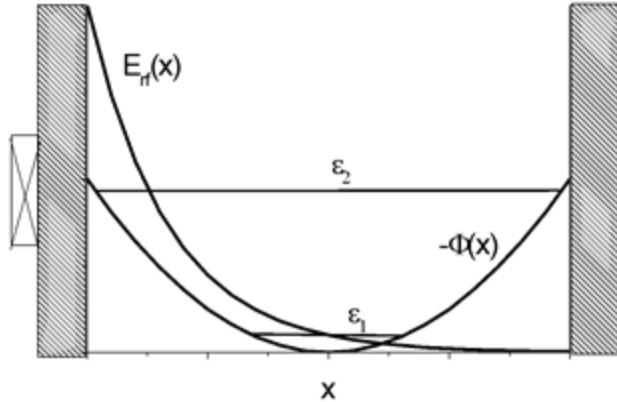
$$\varepsilon = \mathbf{M}\mathbf{v}^2/2 - e\phi(\mathbf{x})$$


Experimental EEDF's at different positions, capacitively coupled RF discharge in argon  
0.03 Torr, 13.56 MHz;

Exp.: V. A. Godyak and R. B. Piejak, APL 1993

PIC: V. A. Schweigert, *et al*, Appl. Phys. Lett. 69, 2341 (1996).

# EXAMPLE: NONLOCAL ELECTRON KINETIC EFFECTS IN INDUCTIVE DISCHARGE



rf electric field

$$\frac{d^2 E_y}{dx^2} + \frac{\omega^2}{c^2} E_y = -\frac{4\pi i \omega}{c^2} [j(x) + I \delta(x)]$$

$$J_y(x) = \frac{e^2 n_{e0}}{m} \int_0^\infty \int_0^\infty G(x, x') E_y(x') dx' + \int_x^L \int_0^\infty G(x', x) E_y(x') dx'$$

$$-\frac{d}{d\epsilon} (D_\epsilon + \overline{D_{ee}}) \frac{df_0}{d\epsilon} - \frac{d}{d\epsilon} \overline{V_{ee}} f_0 = \sum_k \left[ v_k^* (u + \epsilon_k^*) \frac{\sqrt{(u + \epsilon_k^*)}}{\sqrt{u}} f_0(\epsilon + \epsilon_k^*) - \overline{v_k^*} f_0 \right],$$

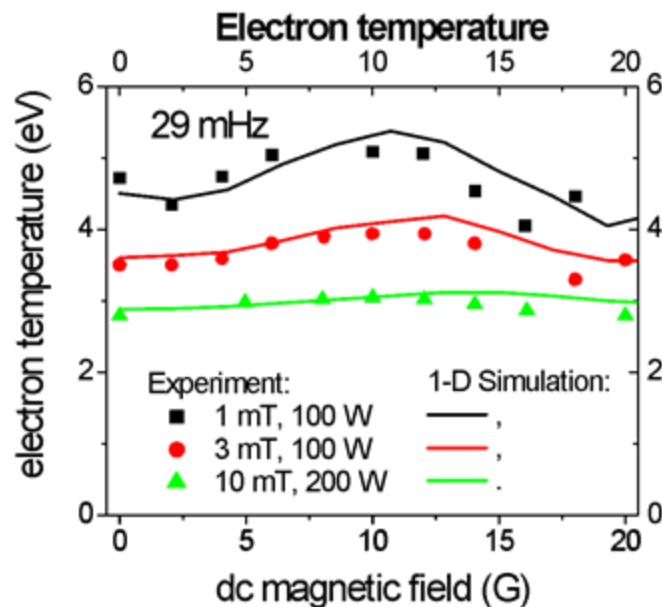
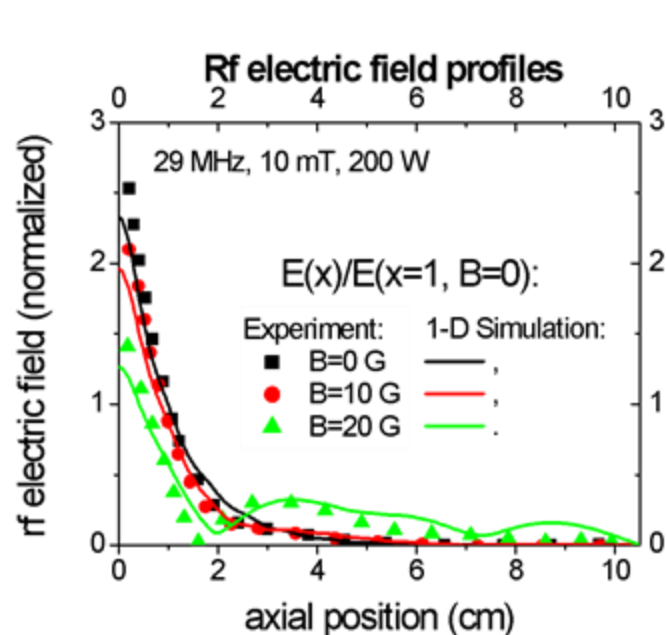
$D_{ee}$   $V_{ee}$  are from the electron-electron collision integral,  $v^*$  is inelastic collision frequency,  $D_e$  is energy diffusion coefficient is a function of the rf electric field  $E_y$  and the plasma potential  $\phi(x)$ . upper bar denotes space averaging with constant total energy .



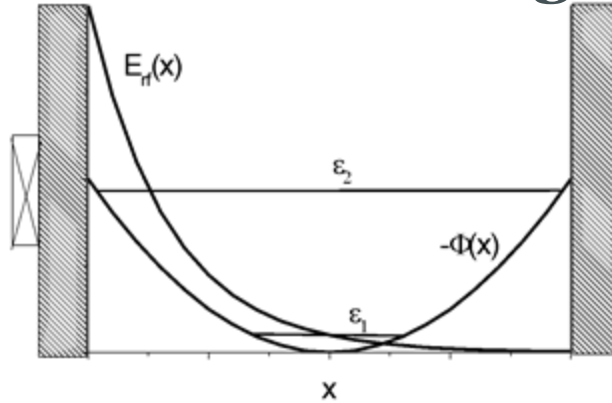
# Comparison with Experiment for ICP With External Magnetic Field

Experiment: V.A. Godyak and B. M. Alexandrovich, Phys. Plasmas **11**, 3553 (2004).

Simulations: O. Polomarov *et al*, IEEE TPS **34**, 767 (2006).



# Inductive Discharge



The transverse rf electric field is given by

$$\frac{d^2 E_y}{dx^2} + \frac{\omega^2}{c^2} E_y = -\frac{4\pi i \omega}{c^2} [j(x) + I \delta(x)]$$

The electron energy distribution is given by

$$-\frac{d}{d\epsilon} D_\epsilon \frac{df_0}{d\epsilon} = S^*(f_0),$$

# Nonlocal Conductivity

- **PIC code is inefficient: limited by electron time step,**
- **while discharge develops at ion time scale =>**
  - **implicit description of the rf electric field**
  - **solved by spectral method**

$$J_y(x) = \frac{e^2 n_{e0}}{m} \int_0^L G(x, x') E_y(x') dx' + \int_x^L G(x', x) E_y(x') dx'$$

Nonlocal conductivity  $G(x, x')$  is a function of the EEDF  $f_0$  and the plasma potential  $\phi(x)$ .

# Nonlocal, nonlinear electron kinetics

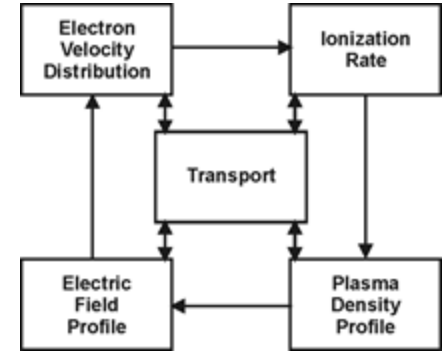
Formation of non-Maxwellian, non-uniform,  $f(v,r,t)$  in self-consistent electric field strongly coupled to plasma density profile through ionization and transport for realistic discharge plasmas.

$$\left[ \frac{\partial}{\partial t} + (\mathbf{v} \nabla) - e\mathbf{E} \frac{\partial}{m \partial \mathbf{v}} \right] f = \sum_k \left[ v_k^* \frac{\sqrt{u'}}{\sqrt{u}} f(w + w_k^*) - v_k^* f \right] + St_{ee},$$

$$\frac{\partial n_i}{\partial t} + \nabla(\mathbf{v} n_i) = v_{iz} n_i, v_{iz} n_i = \int_I f(u) v \sigma_{iz} d\mathbf{v}$$

$$\nabla \mathbf{E} = 4\pi e(n_i - n_e)$$

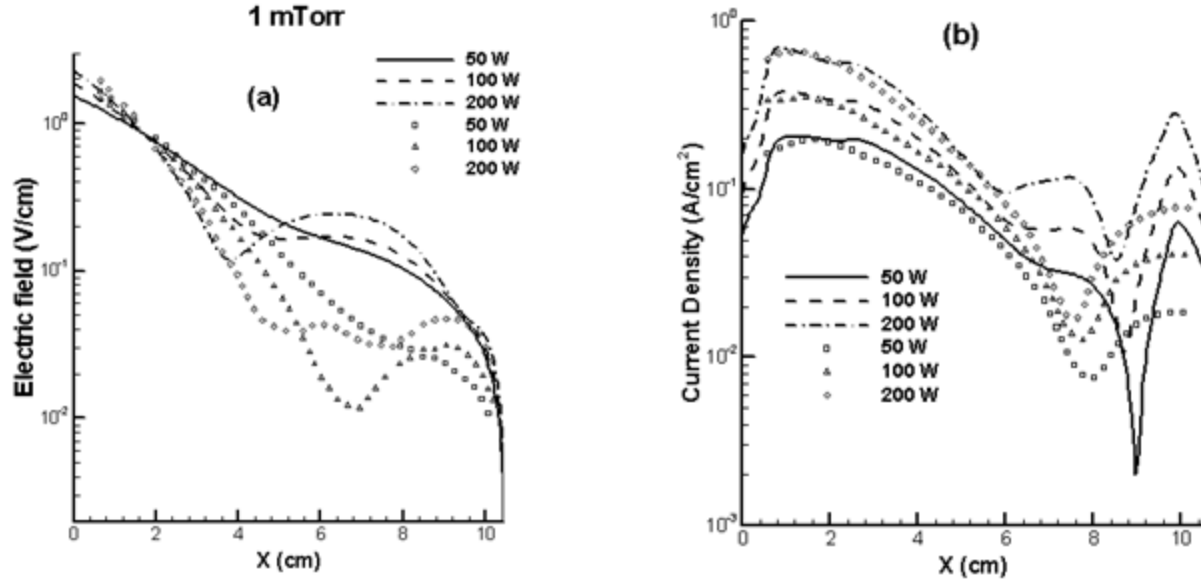
$$M_i \left( \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \nabla) \mathbf{v} \right) = e\mathbf{E} - \frac{\nabla p_i}{n_i} - (v_{ia} + v_{iz}) \mathbf{v}$$



Igor D. Kaganovich et al, Phys. Rev. Lett. 1999, 2000, 2002, 2009, 2012, 2013;  
Valery Godyak et al, Phys. Rev. Lett. 1990, 1992, 1996, 1998, 1999.

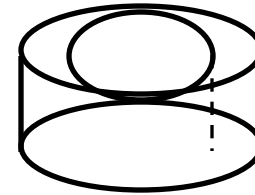
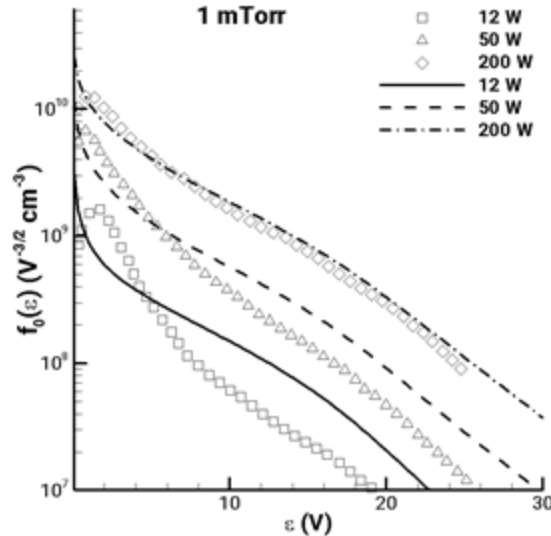
# Comparison with Experiment

$$\frac{c}{\omega_{ep}} < \delta < \frac{V_{Te}}{\omega}$$



Experimental data (symbols) and simulation (lines)  
 (a) RF electric field and (b) the current density profiles for a argon pressure of 1 mTorr.

# Comparison with Experiment



$R=10\text{cm}, L=10\text{cm},$   
antenna  $R=4\text{cm}$

EEDF simulated (lines) and experimental data (symbols) for 1 mTorr. Data are taken from V. A. Godyak and V. I. Kolobov, *Phys. Rev. Lett.*, 81, 369 (1998).